Non-Cataracting Ball Mill Study . . .

Prompts J. F. Myers to question certain grinding theories.

F. C. Bond replies to five points raised.

J. F. MYERS

Definition. A non-cataracting ball mill is so operated that the balls are *not* thrown inward at the top of their paths. The result is to maintain, in the mill, a quiescent pool in which classification will occur, at a limiting size, corresponding to the pulp density. If the pulp density is the same as that carried in an external conventional classifier, the mill discharge will have the same limiting size, as would overflow such an external classifier. This is called IMC (In Mill Classification).

Operation is controlled by the proper relationship of the mill speed, ball size, ball volume in the mill and pulp density. This kind of operation is typified by the primary ball mill operations of the Tennessee Copper Co. at Copperhill, Tenn., and at the Cleveland-Cliffs Iron Co.'s operations near Ishpeming, Mich.

This type of operation has achieved these unusual results:

1. A major reduction in power input.
2. More grinding capacity for metallurgical purposes.
3. A major reduction in grinding media.
4. Minimized "overgrinding" to enhance metallurgical recovery.
5. Increased the rate of mineral flotability by 20 to 30%.

This type of operation has received considerable publicity. The most recent detailed publication appeared in *Engineering & Mining Journal,* June 1953.

A complete explanation of the operation, based on factual evidence and theoretical concepts has not yet been presented, although many misconceptions of the operation have arisen. At the outset, we should realize that this is a process, and not a result achieved by any single factor involved.

Some Explanations Offered

Some observers have considered the big 10-ft inside diameter slow-speed ball mill as the crux of the situation and have viewed it as simply another conventional ball mill slowed down to give power efficiently as recommended by Hardinge and Ferguson.[1] Their idea is perfectly sound, and it was the basic reason for purchasing the big slow-speed mill at Tennessee. Performance of the new large mill soon showed that some entirely different factors were involved, upsetting certain conventional concepts about grinding.

Some feeling exists that success of the process is due to the very special relationship of mineral to gangue in the Tennessee ore. However there are many ores with the same characteristics all requiring some degree of operating adjustment in the crushing and grinding circuits.

Some have approached an explanation by applying the Schuhmann law[2] which demonstrates that log-log plots of comminuted minerals under laboratory conditions cannot have their slopes changed. Commercially ground ores which usually constitute a mixture of minerals with varying specific gravities produce different slopes. This introduces a complication which is later discussed and illustrated in this article.

Another explanation of the method has come from adherents to the Bond[3] Third Theory and formula, in which the whole evaluation of metallurgy is based on the size of material passing the 80% passing mark.

Finally, there are those who place great weight on the hydraulic classification feature of the process which gives a high depression factor, or, in other terms, a high settling ratio between mineral and gangue in the coarse sizes.

It seems to the author, that a definition of what we are trying to do is very much in order. The sole purpose of comminution in a metallurgical process, is to free or release the economic minerals from the gangue minerals at the point dictated by economics. Grinding the free or released economic minerals beyond this point is a waste or misapplication of power. Admittedly, cutting off the power application at this point in a metallurgical grind does not generate as much particle surface or as fine a material at the 80% passing mark, which is non-essential if the mineral and gangue are liberated.

The reader may say that there is nothing new in this concept; still all of us are aware that in arriving at any given mesh size for mineral liberation, by *conventional means,* a lot of mineral of process size, is ground farther than need be. This has led to the much abused term of "overgrinding." The non-cataracting mill, close circuited with a hydraulic classifier was designed for metallurgical purposes *only,* as defined above.

Unfortunately, development of the process covered a period of several years. There is no question about the results reported at the start, Column AA, Table I, nor is there any question about the results as published in June 1953, Column FFF. A true evaluation of the various steps taken during these years cannot be made. There were no sharp cutoff points between them other than in a general way.

At Tennessee the influence of changes in rod mill development, overlapped and effected changes in classifier and ball mill study as both studies were going on at the same time. Those familiar with grinding operations, realize how much time it takes to balance a ball and rod charge after a change is made. It will remain for other investigators, using parallel equipment, to bring the various factors into sharp focus for proper evaluation. If the reader will keep this in mind, then the ensuing discussion will have some degree of merit.

A bit of history is well worth reviewing. Some 53 years ago open circuit grinding became obsolete be-

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THREE YEARS AGO an article, **"The Remarkable Case of the Copperhill Ball Mill,"** *E&MJ* June 1953, touched off a stimulating debate on various fundamental phases. Prominent mill men in Canada and the United States participated, including the authors of this article. Subsequent review of ideas expressed in this exchange of views has prompted J. F. Myers to re-open the subject, and Fred Bond to reply to questions raised. We hope that this new interchange of views will stimulate new and practical interest in the not-too-completely understood art of grinding.—The Editor

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cause of invention of the Dorr classifier. Closing the circuit with classifiers increased mill capacity and reduced power.

Long tube mills kept a ball charge tumbling to break a minor amount of unground ore as it travelled the full length of the mill. In so doing, a tremendous amount of sufficiently ground mineral grains was subjected to further reduction, consuming power and mill capacity.

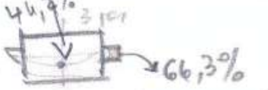
The degree of classification was then only partial, just as it is today, some 53 years later, in conventional grinding circuits. Much of the finished size material was and still is returned via the classifier sand loads back to the grinding mills where it still consumes power and mill capacity.

For those interested in log-log plots from screen analysis of comminuted ores, it is noteworthy that closing the circuit by means of classifiers, bent the plots in the course sizes and steepened the slope in the finer sizes. Bond[4] records this fact in his Montreal and Chicago papers.

In these same papers, Bond also records the fact, that as grinds get finer, conventional classification becomes progressively less efficient and that the log-log plots have flatter slopes accordingly. It is generally acknowledged that as classification becomes inefficient, so does grinding efficiency, and the capacity falls off.

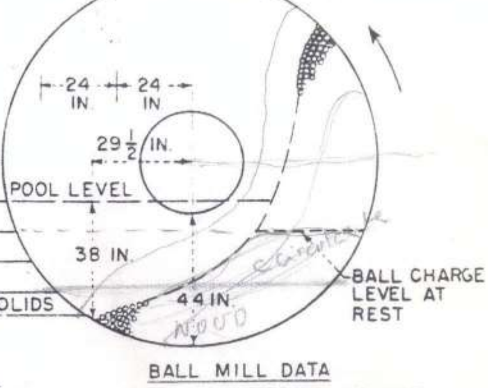
It is only within the last few years that any fundamental improvements have occurred in classification pertaining to primary grinding circuits, namely hydraulic machines cyclones and IMC.

FIG. 1. LONDON BALL MILL CLOSED CIRCUITED WITH HYDRAULIC CLASSIFIER, JANUARY 1953 FEED TO ROD MILL 2,250 TONS



POOL SAMPLES 3 FT FROM TRUNNION OVERFLOW

	% +65	% −200	% SOLIDS	
	6.5	51.7	46.2	POOL LEVEL
10 IN.	20.4	24.5	63.5	
10 IN.	28.1	19.3	70.9	
10 IN.	30.4	19.7	71.1	
	72.4 TO 74% SOLIDS			
HYDRAULIC CLASSIFIER OVERFLOW	7.6	54.4	39.2	
MILL DISCHARGE	23.9	23.9	66.3	



CLASSIFYING POOL IN MILL
7 FT WIDE, 11 FT LONG

BALL MILL DATA
BALL LOAD - 24.5% OF MILL VOLUME
BALL LOAD - 31.7 TONS
RPM - 15.0
MAKE UP BALLS - 1.0 IN.
MEAN DIA. BALLS - 0.7 IN.
APPROX. 48 LAYERS OF BALLS ASCENDING

What Really Happens

And now the reader has one of the basic principles underlying the ability of the non-cataracting ball mill to reduce power and gain in capacity, namely; (1) by enhancing the elimination of proper process size ore grains from the ball-mill feed at a high depression factor, by means of hydraulic classification and (2) elimination of ore grains of proper process size, as fast as they are generated in the ball mill by means of the classifying pool in the mill itself.

Many technicians may question the claims of the effectiveness of the classifying pool in the ball mill. Under the direction of F. M. Lewis, mill superintendent of the Tennessee Copper Co., a complete study was made of the pulp conditions of the pool as it is normally operated.

Samples of the pulp were removed from the pool, both laterally and logitudinally at every ten inches of pool depth.[5] Fig. 1 shows a summary of the results existing three feet from the discharge opening. At 46.2% solids, this is a creditable job of classification.

It is to be noted that the non-cataracting feature of the ball charge provides a quiet, undisturbed pool surface, which is 6 ft wide, and 10 ft long. This is essentially the equivalent of a 6-ft classifier. No pulp is visible exiting from the ball charge above the pool level, consequently, there must exist a rising current of pulp under the pool surface, as it exits from the ball charge. This effect is similar in action to the rising current effect in a hydraulic classifier.

A major power and capacity benefit is derived from the high depression factor produced by the classification in the mill pool with an assist from the hydraulic classifier. The gangue is not ground so fine, and it is essentially freed from the mineral constituents.

Let's define the term "depression factor." We are all familiar with the fact that in an ore containing two or more minerals of different specific gravities, that due to classification the heavier minerals grind finer than do the lighter minerals. This is expressed in the literature in various ways, such as ratio of settling, differential separation, differential grind, etc. None of these terms can be expressed in a single mathematical figure to designate the extent to which the phenomenon has occurred.

The term depression factor was coined to express the number of mesh sizes between the largest particles of essentially free gangue in relation to the largest essentially free particles of a heavier mineral resulting from classifying action.

Illustration. If the largest particles of gangue rock are 48 mesh in size and the largest particles of sulphide mineral are 100 mesh in size, then the depression factor is 2.0. Each mesh represents a factor of 1.0.

A better understanding of the depression factor can be gained from log-log plots, see Fig. 2. In this case there is a two mesh spread, at 100% passing between the mineral and gangue curves. Hence there is 2.0 depression factor.

Due to imperfections in classification and due to the arbitrary cut off point dictated by economics to limit the grind, there are always some "chatty" particles of a minor nature. These are usually ignored in setting up the log-log curves between mineral and gangue.

The depression factor of 2.0, in Fig. 2 is that produced with the non-cataracting ball mill with an assist from the hydraulic classifier closed circuited with it, on ore from Tennessee Copper's London mill. A depression factor as high as 2.5 has been reported. Fig. 3, is the log-log plot of mineral and gangue on London ore produced with a conventional ball mill and conventioal classifier. The depression factor in this case is only 1.4.

Subsieve size analyses are available in the data that comprised the Fig. 2 curves and these also have been plotted. The characteristic "cross over" point is shown at which point there is no difference between the per cent passing of the mineral and gangue. This is always true on sulphide ores.

Very little information is available in the literature relative to subsieve-analysis. Yet by and large the subsieve sizes constitute the greatest per cent weight in most metallurgical processes. The crossover point is controlled by

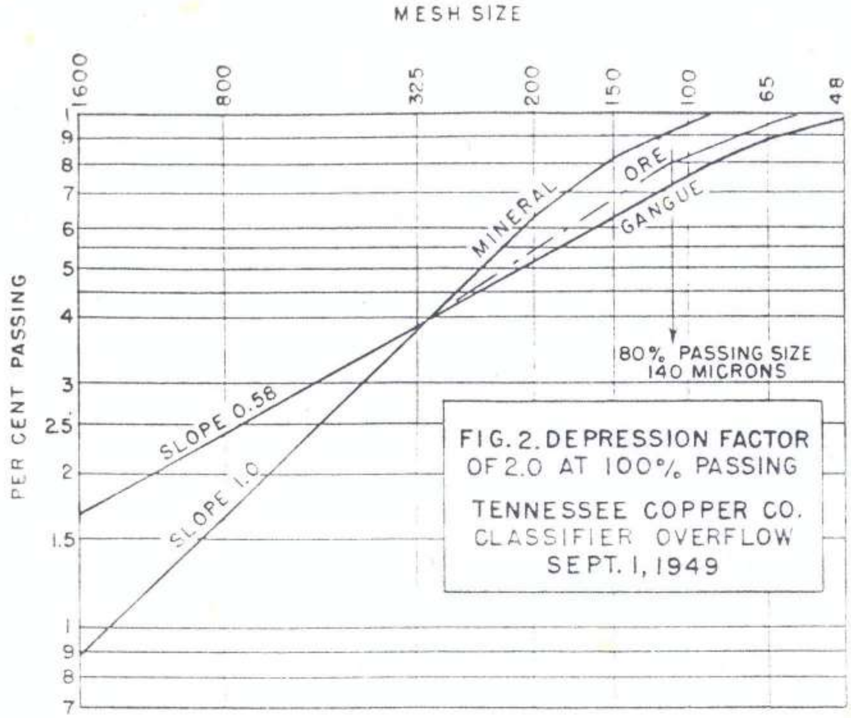


FIG. 2. DEPRESSION FACTOR OF 2.0 AT 100% PASSING

TENNESSEE COPPER CO. CLASSIFIER OVERFLOW SEPT. 1, 1949

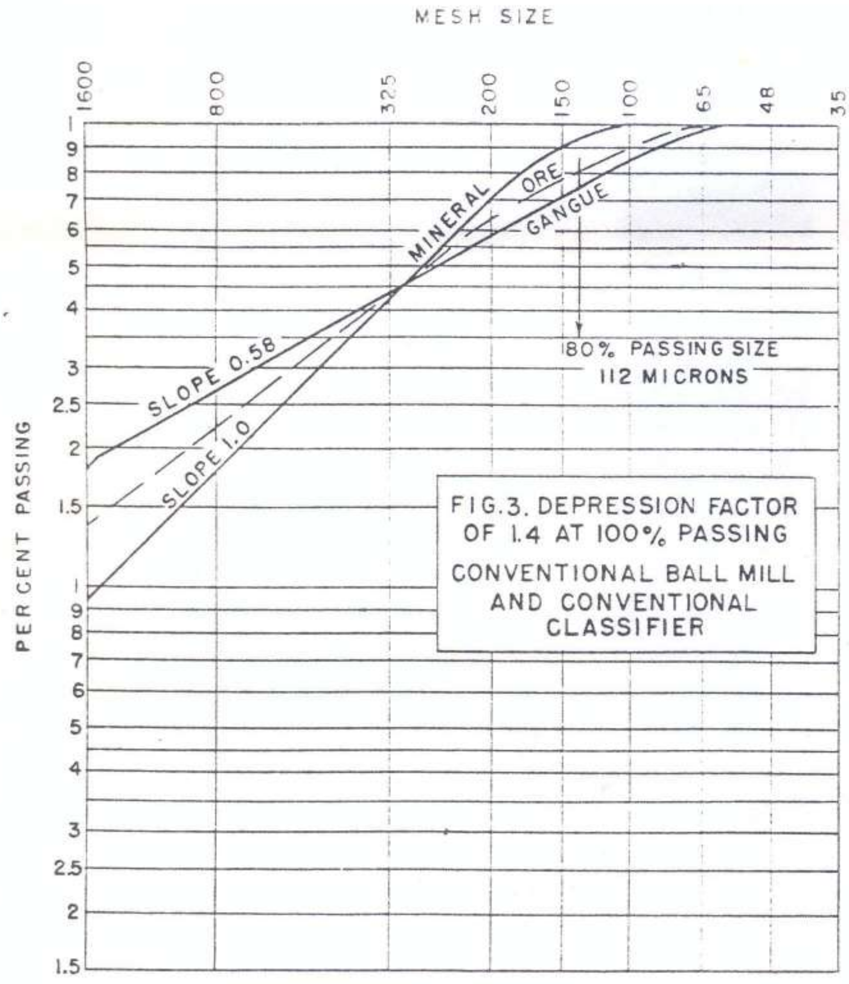


FIG. 3. DEPRESSION FACTOR OF 1.4 AT 100% PASSING

CONVENTIONAL BALL MILL AND CONVENTIONAL CLASSIFIER

classification and technicians should know more about this matter than they do.

Another power reduction of approximately 13.5%, developed when the volume of the ball charge was dropped from 45% to 24-28%. Also, a nice gain in daily capacity was obtained. This was recorded by F. M. Lewis in *E&MJ*, June 1953.

The ball charge at Tennessee is provided by the daily addition of one-inch make-up balls. As it stands today, we understand very little about small balls in tumbling mills. This fact came pretty close to being the undoing of the process at Tennessee in the early stages of its development. Formulas failed and mental conceptions of ball action, established by operating experience, were useless. A few facts can be stated about a charge of one-inch make-up balls, as they have been disclosed at Tennessee.

1. In mills up to six feet in diameter, at conventional speeds, the ball charge accomplishes its work by balls cataracting, cascading, rolling and attrition.

2. In mills up to six feet in diameter, at conventional speeds, the pulp dilution in the mill, as in any other conventional mill, should be from 70 to 74% solids. In a 10-ft mill, some classifying pool is essential for improved results over small diameter mills. It should be much less than 70% solids. As low as 35% solids has accomplished good results. 4

3. In six-foot mills, at conventional speeds, grinding is roughly proportional to power input. In the 10-ft mill with a ball charge of 45% of the mill volume, after the power reaches 410 hp little or no more grinding takes place, as the power input is increased, by speed.

4. In any size mill, fast or slow, a charge of one-inch make-up size cannot operate successfully except with a small circulating load. Too much unground pulp at the toe of the charge dampens or cushions the little balls' action drastically. !

At the moment, the ratio of mill diameter to the diameter of the make-up balls, seems to be the governing factor as to whether the mill cataracts its balls.

With the Tennessee six-foot mills, the ratio is 67:1, and the balls do cataract. The ratio is 120:1 in the 10-ft mill, and the balls do not cataract.

Technicians are well aware of the benefits of small balls. Ball size is determined by the largest pieces of ore encountered. A little oversize or coarse feed particles can be an expensive cost item. Variation in the hardness of the ore is a contributing factor. Usually balls 2-in. or larger are pretty much conventional practice, although 1½-in. is used in a few operations.

This brings us to the rod mill operation that produces the feed for the non-cataracting ball mill. At Tennessee, this unit is the prima donna of the process and it is probably the most pampered rod mill in the industry. No rod mill manufacturer quite agrees with design features, nor do technicians agree with the mode of operation. The unit has been under test conditions since 1929.

The rod mill has a high ratio of length to diameter, a high dilution, high speed, high rod charge, vertical end liners, and a special grooved shell liner, with a drum feeder and high-level discharge. These factors may all improve the grinding efficiency. However, it is the author's opinion that the most important single factor in improving the grind is the practice of removing worn rods weekly, so that the mill never contains broken rods, flat rods, pointed rods or any other scrap metal pieces.

The Bond Formula

For the benefit of some of the younger and less experienced readers, a bit of explanation relative to what the Bond formula is and how it works might be of interest. There is no ...ticular mystery as to how formulas ...e generated to fit a given set of conditions or factors. It follows that a formula can fit only one set of factors. Some universities give courses relative to the subject.

In this particular case, Bond took a lot of grinding results that were available to him and, after culling out extreme cases that were not consistent with the better types of operations, he struck an average of the factors pertinent to the operations, and developed a formula to fit these factors and conditions. Therefore, the use of the formula gives *only* the power W in terms of the efficiency of the combination of factors upon which the formula for W was predicated.

It must be constantly borne in mind, that in solving for W, one arrives at the power value, consistent with average operation of 7½-ft I.D. ... mills and 6-ft. open circuit rod mills, etc., as it existed in the grinding technique prior to 1950. It, therefore, becomes perfectly obvious that something has to be subtracted from W to take care of efficiency gains, as the art of comminution progresses and more efficient machinery develops.

To illustrate this point, large diameter mills are more efficient than small mills. Bond gives us the correction figure for this better efficiency. Unfortunately, he does not have, or at least never has published, any corrective figures for the efficiency gains on the other factors such as ball size reduction, hydraulic and cyclone classifications, mill speed, etc.

It then becomes obvious that without the correct value of W, one can get mixed up and confused in trying to make comparisons by means of the Work Index, Wi in the formula.

Another source of error lies in the accuracy of the small sample used for making the grindability tests. The accuracy of W depends upon the accuracy of the sample. This is hard to arrive at when an orebody contains various degrees of hard and soft ore. For the most accuracy, grindability figures on at least three mesh sizes should be established to be certain what one is dealing with in relation to bonds of adhesion and cohesion. This also helps in interpolating the grindability between meshes.

The author is amazed at the meager information available for technicians to design circuits. Errors as high as 48% have been reported. On rising metal prices this is wonderful, providing the error was on the plus side. Nevertheless it constitutes an engineering error. When the error is on the minus side, that is bad.

Role of the Bond Formula

Many might ask what good is the Bond formula in solving for power requirements if all of these corrective figures must be taken into consideration. Where does one find the corrective figures? How does one know if he has all the corrections? The fact is, it is not a happy situation, but the author would like to point out that prior to the Bond formula, technicians had nothing to go on and were completely dependent upon manufacturers' representatives to give information. The record does not indicate that that was always a happy situation either.

In using the Bond formula, many technicians fail to realize that they are taking into consideration only 80% of the ore: namely, that portion passing the 80% passing-size mark, as that is what the formula is based on. Thus it is presumed that the other 20% will fall into place and behave properly. This it usually does, providing nothing happens to disturb the normal distribution of the mineral and gangue relationship created by conventional classification, or that no unusual quantities of irregular shaped particles exist. For this reason the formula has no direct application when cyclones or hydraulic classification are involved or on small balls, etc. It will be found accurate only when the conditions of the problem to be solved are close to the base line factors of the formula.

Importance of the Bond Formulas

In the author's opinion, the Bond formulas are among the most important developments of the decade in the art of comminution. Proper application is an important contribution to our "know how." The same applies to the Bond rod mill formula. Here again one must consider the fact that the formula deals only with the 80% passing size. To measure relative efficiency on this basis only does not tell the whole story. Tramp oversize in the 20% of the ore plus the 80% passing mark plays a vital part in the ball mill operation and overall operating cost.

That a mill can be slowed down, the ball charge reduced, etc. and still have the same or better capacity, is apparently more than experienced technicians are willing to accept regardless of the contention that "something new has been added."

The most publicized criticism along this line come from F. C. Bond.[6] Mr. Bond contends that a much smaller, fast conventional mill will produce the same amount of 80% passing size material for the same power. Hence, he and some others, do not believe that the 10-ft mill of the Tennessee Process will handle as much tonnage capacity as if it were running up to conventional speed.

This misconception arises from the lack of understanding and appreciation of the change in depression factor in the 20% of the ore that the Bond formula does not take into consideration.

By again referring to Fig. 3, it will be observed that the 80% passing size, of the ore, is 112 microns, which is essential for mineral and gangue liberation at 100% passing by conventional classification.

The value of P is therefore 112 in the formula.

In Fig. 2, it will be noted that the same degree of liberation occurs with an 80% passing size at 140 microns on the ore due to the hydraulic classification and non-cataracting ball mill at a depression factor of 2.0. Consequently any attempt to evaluate the non-cataracting ball mill with an 80% passing size greater than 112 microns is obviously incorrect as the mineral and gangue liberation would not take place at the 100% passing size, by conventional mills and classifiers. At the liberation point the mineral curve must stay pegged at 100 mesh at the 100% passing size. Incidentally, cyclones have theoretically about the same ability to create high depression factors close circuited with the proper ball mill action. (μ)

The author feels that some constructive discussion of this matter is in order, due to the potential possibilities of cyclone classifiers, hydraulic classification and IMC (In Mill Classification). Metallurgical engineers are concerned with mineral and gangue rock liberation at the 100% passing size of log-log plots, at a minimum expenditure of power. What happens at the 80% passing size, or any other point, has nothing to do with metallurgical results. It has been shown that classification can and does change the slope of log-log plots of comminuted ores by changing the distribution of various minerals in the ore. Individual minerals do not change slope which fact is in accordance with the Schuhmann concept. It must follow that as conditions arise to change the slopes, then, for formula purposes of the Bond type, different passing points must be established. The 80% point is purely arbitrary to suit the condition of slopes established by fast conventional mills, close circuited with conventional classifiers.

Mr. Bond's ideas are purely hypothetical, as there does not exist in the industry a primary grind with a fast 10-ft mill operating, with 1-in. balls. Table 1 reviews published figures, as disclosed by the Tennessee Copper Co. The tonnage figure in Column FFF was as published in 1953. Any way one looks at it, that is a lot of mineral and gangue rock liberation for the power applied. Tonnage has been materially increased as of this date.

Mr. Bond also contends that 1-in. balls should be considered as "conventional," although at the time he established his basic factors upon which the formula was devised, there did not exist any conventional primary mills, of any size, using 1-in. make-up balls, nor do any exist today.[7] There are a few operations using 1½-in. balls. In general, rod mill operation has improved greatly during the past several years and the feed to ball mills is generally finer. One would expect the trend to be to smaller balls, as time goes by, as the benefits powerwise and in tonnage capacity are well known.

A 30% Saving in Power

The point Mr. Bond wishes to make is that purchasers of ball mills can save capital expense in buying fast conventional mills rather than big slow-speed mills, although he willingly admits that the non-cataracting mill process is highly efficient. Over the years there has been a lot of this kind of talk relative to grate mills, slow speed mills, etc. Strange enough, there is a total lack of comment in the metallurgical literature relative to powerhouse capital cost. This will vary in different locations, but a fair figure seems to be $200 per kw of installed capacity. The power saving of the non-cataracting ball mill and 1-in. balls, properly operated for a high depression factor, offers something in the order of 30%. This, reflected back to the cost of power plants or in power

Table 1—Grinding Data

	Conventional ball mills with conventional classifiers AA	Non-cataracting ball mill with hydraulic classifier FFF
Tons per 24 hours	1,250	2,250
Input, hp	387	370
Kwh per ton	5.54	2.97
Per cent of critical speed	86.4	62.1

contracts, is interesting to contemplate.

The reader may well ask, why is all this detailed discussion of Mr. Bond's viewpoint so important. It is simply this. The author believes that the factual evidence supports the contention that any 10-ft diameter mill, with a good rod mill operation ahead of it, can convert to the non-cataracting process and in so doing gain a little capacity, reduce operating costs and improve the metallurgical results. Mr. Bond does not think that slowing down the mill will grind as much ore to the same amount of 80% passing size. The author contends that it will grind as much ore or even a little more to the economical degree of mineral and gangue liberation at 100% passing. Admittedly the author's viewpoint would not apply to industrial grinds such as cement aggregate, cement plants, etc. where the material to be ground is of a homogeneous nature.

The foregoing indicates that hydraulic classification plays a very important part in the non-cataracting mill process. Just how much of this occurs in the classifier and how much in the mill pool, is not known.

During the development period, the hydraulic classifier was out of commission for several days on two occasions. Both times the plant was kept operating successfully, depending only upon the classification in the mill pool. At these times the dilution in the pulp existing from the ball mill was the same as normally overflowed the classifier.

"In Mill Classification"

"In Mill Classification" (IMC) is a factor that has a lot of potential possibility, either alone or in conjunction with hydraulic classifiers or cyclones.

It will be remembered that the process was devised to minimize oxidation in the grinding circuit, and hence, enhance the recovery and grade of concentrate. Space does not permit a discussion of this subject, but it can be stated that the process has been extremely successful in this respect. The non-cataracting ball mill process has much to recommend it to those who have easily oxidized minerals to grind and process.

The question arises as to whether other ball mill diameters can be converted to the non-cataracting type. In so far as the author knows, the crux of the situation lies in the ratio of mill diameter to ball diameter. Something in the order of 120:1. It will remain for further investigation to clarify this question as well as to evaluate and work out other details.

Operating Observations

In the event that the non-cataracting ball mill is close circuited with any type of classifier or cyclone, it seems to make little difference whether or not the rod mill discharge feeds direct to the non-cataracting ball mill or to the auxiliary equipment, provided that the dilution in the mill is low enough.

Small balls, ¾ to 1¼ in., are limited in ability to break feed particles coarser than 8 to 10 mesh, depending upon grindability.

Proper means must be provided at the discharge opening to prevent tramp oversize exiting in the pulp.

The lifters on the liners should not be higher than the diameter of the make-up balls, otherwise the balls will pocket and reduce the capacity. The lifters should be close enough to minimize slip on the shell.

Obviously the non-cataracting principle applies only to trunnion overflow mills.

At the present time, mill speeds from 58 to 64% of critical seem to be best.

The diameter of the trunnion discharge opening should be designed to exit the classified pulp in the mill.

References

[1] The Effect of Mill Speed on grinding Costs, Hardinge and Ferguson, AIME Transactions, Vol. 187.

[2] Principles of Comminution, R. S. Schuhmann, AIME T.P. 1889, 1940.

[3] Third Theory of Comminution, F. C. Bond, AIME Transactions, Vol. 193.

[4] Crushing and Grinding Calculations, Allis-Chalmers publication, F. C. Bond, Montreal 1954 and Chicago 1955.

[5] Sampling report in author's file.

[6] F. C. Bond, consulting engineer, Allis-Chalmers Co.

[7] Correspondence in author's file.

Mr. Bond Answers Questions Raised by Mr. Myers

FRED BOND

MR. MYERS HAS MADE an interesting presentation of material favoring the non-cataracting ball mill, and many points in his discussion are well taken. However, his treatment of the Third Theory of Comminution, and the application of the work index, shows that the subject is not fully understood.

His general objections to the Third Theory calculations seem to be about as follow:

1. The 80% passing size criterion does not completely define the size distribution.

2. The equations are empirical and can fit only one set of factors. In other words, the theory is not generally applicable.

3. He is not clear about the function and use of correction factors.

4. The 80% passing size and work index do not take into account the metallurgical results obtained from a grind. These depend upon such items as the type of breakage and the "depression factor."

5. The samples used for grindability tests may not be representative. Tests at several mesh sizes are desirable.

The Third Theory states that the useful work done in crushing and grinding is inversely proportional to the square root of the diameter of the product particles, and hence is directly proportional to the new crack length formed, or to the square root of the new surface area formed. It is in disagreement with the older Rittinger Theory, which states that the work is directly proportional to the diameters of the product particles. It is also in disagreement with the Kick Theory, which states that the work is proportional to the reduction in volume of the particles concerned.

Any analysis of the work done in crushing and grinding requires the application of some theory of comminution, and as Mr. Myers states, no practical method was developed until the discovery of the Third Theory. On the other hand, the Third Theory does not automatically solve all crushing and grinding problems. It is merely a useful tool, with skill and knowledge needed in many applications.

The Third Theory is the logical result of the combination of strain energy and the modern concept of brittle breakage. Stress is applied to the rock and it is deformed, absorbing strain energy. Crack tips are formed when the stress locally exceeds the breaking strength. With the formation of the crack tips the strain energy flows, splitting the rock, and the energy is released as heat. The energy required to break is proportional to the length of the crack tips formed.

The Third Theory has a more complete theoretical basis than the two old theories. The Rittinger Theory rests on the erroneous assumption that the surface energy of solids equals the energy required to break, or is proportional to it. The Kick Theory applies the strain energy of deformation to the breakage of cubes or other equi-dimensional particles, without reference to the formation of crack tips as the essential process of brittle breakage.

No direct method for the measurement of the crack length of broken rock exists; however, it is mathematically equal to the square root of one-half of the surface area, and hence is inversely proportional to the square root of the particle diameters. This relationship is used in applying the Third Theory.

1. The 80% passing size is practical.

The size 80% passes had been used by Prof. Taggart in his *Handbook of Mineral Dressing*, and was selected as a practical, readily obtainable, and reasonably accurate criterion of the size of a crushed or ground product. It is a selected point on the plotted size distribution line, and represents the 20% of the ore above that point just as accurately as it does the 80% below. It does not define the slope of the line above or below, in accordance with the first point cited as an objection by Mr. Myers. The slope is a function of the work index, and any change in slope is equated by a change in the work index value. If the plotted product slope is increased, the increased efficiency will be shown by a decrease in the work index. The wide application of the Third Theory to crushing and grinding problems would have been impossible without the use of a passing size criterion.

When P equals the size in microns which 80% of the product passes, F is the size 80% of the feed passes, W is the kwh input per short ton, and Wi is the work index, then in Equation 1:

$$W = \frac{10\,Wi}{\sqrt{P}} - \frac{10\,Wi}{\sqrt{F}}$$

This is the basic equation and mathematical statement of the Third Theory; any one of the four quantities concerned can be obtained from it when the other three are known. It has been found to agree with actual crushing and grinding conditions over the entire size reduction range. It is not limited by any operating conditions, but has a general application to the entire field of comminution.

The work index is the reference standard, from which the work required to reduce from any feed size to any product size can be found by Equation 1. Numerically, it is the kwh required to reduce a short ton from theoretically infinite feed size to 80% passing 100 microns, equivalent to about 67% passing 200 mesh. As the work index on any material decreases the mechanical efficiency of reduction increases.

2. Third Theory development is not empirical.

In denial of the second objection cited: there has been nothing empirical in the development of the Third Theory as outlined above, unless confirmation of the agreement of theory with facts can be so called. The efficiencies of different reduction machines, and different operating conditions on the same ore can be compared directly by calculating the work indices from transposed Equation 1.

However, the transformation of laboratory crushing and grinding test results into work index values is done empirically in order to make the work index as determined from laboratory tests correspond to the average of actual plant operations. An empirical formula which represents the average of all the data available in that field, was developed for each type of laboratory comminution test. With these formulas new installations can be planned from laboratory tests, and individual plant operating efficiencies can be compared with the average efficiency as shown by a laboratory test on that ore.

The efficiencies of different plants operating on different ores also can be compared with each other, using

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