

Flotation as an Operation of Macromolecular Mass Transfer

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ABSTRACT

The present work presents a new theoretical interpretation of the flotation process, which can also be applied to other mineral processing physical unit operations. This interpretation is part of the 'Modelo Operacional' (Operational Model) that was developed as an attempt to establish a new theoretical basis for mineral processing unit operations. In the case of flotation, a book was edited by one of the authors in Brazil (*Mass Concentration by Froth Flotation*, in Portuguese and Spanish).

The model establishes a link between a natural phenomenon (migration of particles from the pulp to the froth) and a real industrial phenomenon (mass concentration by operational interference). In this model, flotation is viewed as a phenomenon of macromolecular mass transport (of particles) between froth and slurry phases. Mechanisms for mass transport and particle flow between phases as a function of the hydrophobic potential in each phase are discussed. By establishing the so-called *equilibrium distribution curve* between phases it is possible to calculate macroscopic balances that are related to metallurgical results, which in turn are a function of the *operational line* that is required by the process itself. The *number of cleaning stages* and their efficiencies can also be determined by simple laboratory-scale experiments.

MACROMOLECULAR MASS TRANSFER

New approach

In chemical engineering, mass transfer is described as the modification of solutions or mixtures of different chemical composition by means of methods that do not necessarily include chemical reactions (Treybal, 1970). Generally these operations are related to the separation of a substance from its components. In the case of *mineral particle* mixtures, separations can be entirely mechanical, as in certain unit operations, such as filtering and classification by screening. Separation of particles based upon differences in density, magnetic properties and also separation of mass by froth flotation, among others, promotes changes in chemical composition between products or separated phases. Therefore, as they include changes in flow composition, those operations can be defined as being macromolecular mass transfer operations.

In an analogous way, macromolecular mass transfer can be accomplished by flotation. Collector reagent is adsorbed into the surface of interest particles, creating or increasing their hydrophobicity. Gas addition to the slurry phase and capture of hydrophobic particles characterises macromolecular desorption, removing mass from slurry to the gas phase and keeping the ascending flow to the froth layer. Water addition to the froth surface (froth washing) drags hydrophilic or less hydrophobic particles back to the slurry phase, characterising a macromolecular absorption and creating a double draining mechanism in the froth phase.

Transport mechanism

On a molecular scale, mass transfer is the result of a concentration gradient that induces the movement of molecules from a region of high concentration to another of low concentration. In flotation, a

particle does not move by itself, but is attached to air bubbles, from slurry to mineralised froth where it is dragged after being lifted to the average portion of froth flux. Desorption continues in the froth phase by macromolecular diffusion in laminar flow (not turbulent), and thereby a continuous concentration gradient from the froth/slurry interface until a froth surface is established. In a steady state the froth/slurry interface is a theoretical representation of contact between phases. When equilibrium is reached, particle transfer finishes and the final interface becomes physically well established, allowing final separation of products or phases.

Gradient and driving forces

The average speed of particles depends on their hydrophobicity. The most hydrophobic ones have a higher affinity with air bubbles, giving rise to stronger froth that will quickly be transported to the surface of slurry. Free particles of a certain size range will respond better to the process. In the slurry phase, the contact bubble/particle is quickly produced by means of mechanic or pneumatic agitation, while keeping the necessary homogeneity of this phase. The speed of macromolecular transfer is very high, mainly in mechanical cells. The mass flow of the substance of interest ($N_x = \text{mass/time}$) can be defined by means of the following expression:

$$N_x = \text{Transfer Coefficient} \times \text{Contact Area} \times \text{Gradient}$$

The hydrophobic difference between particles is the transport gradient, which can be related to particles' individual grade or liberation (and then best reagent adsorption and more hydrophobicity). The transfer coefficient possesses the same flow units divided by the gradient and the area ($\text{mass}/[\text{time} \times \text{area} \times \text{gradient}]$). Let a be the average grade of the substance x in the slurry being fed to the process, and r_f the average grade of particles in the interface between slurry and froth, as illustrated in Figure 1. Knowing that: $N_x = K_p \times A \times (a - r_f)$ where K_p is the transfer coefficient in the slurry phase. The gradient ($a - r_f$) is the driving force (only an analogical supposition) that promotes macromolecular desorption in the slurry phase (particle capture from the pulp to air bubbles). The phenomenon that happens in the slurry phase is called *natural flotation*, or simply *flotation*. In the froth layer, due to their hydrophobicity, particles are spontaneously transported until the point of lowest contact angle with water – or lowest pressure. That is, until the surface of the froth layer comes in contact with the atmosphere.

For the froth phase, the following relation is defined:

$$N_x = K_E \times A \times (c - c_f) = K_p \times A \times (a - r_f)$$

CONCENTRATION **FLOTATION**
FROTH **SLURRY**

where:

K_E is the transfer coefficient in the froth phase, with the same units of K_p

c is the average concentrate froth grade

c_f is the interface particles grade

In a similar way to what was defined for the slurry phase, the gradient ($c - c_f$) is the driving force that promotes particle transfer from the interface to the main froth concentrate flow. The phenomenon that happens in the froth phase is called *concentration*.

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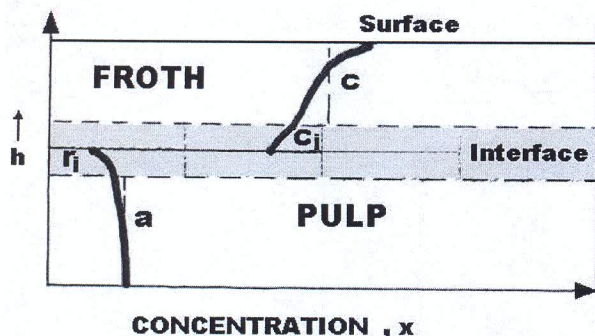


FIG 1 - Gradient and impelling forces. The hydrophobic difference between particles is the transport gradient, which can be related to particles' individual grade or liberation.

Equilibrium

The degree to which an operation approaches equilibrium impacts metallurgic results, circuit characteristics, number of stages, circulating loads, etc. The closer to equilibrium, the more efficient the process will be, with better metallurgic results and lower production costs. In systems comprising two phases (such as slurry and froth) that are not in equilibrium, a spontaneous change due to macromolecular migration takes place, driving the system to the equilibrium state, where further changes no longer occur. After a determined period of time, it is observed that the concentration of the substance of interest is the same throughout a phase, but not necessarily the same for both phases. As previously mentioned, once equilibrium is reached, the uniformity between both phases is different:

- *Slurry*: due to mechanical or pneumatic agitation, this phase is rather homogeneous. The ore grade r^* is generally the same in any vertical section and, in addition, pulp density and particle size distribution remain practically unchanged.
- *Froth*: vertical organisation caused by different particle hydrophobicity occurs, with average ore grade c^* . As $(c^* - r^*)$ is not a true concentration gradient, no further mass transfer between froth and slurry takes place.

Natural phenomenon versus real phenomenon

The natural phenomenon happens 'to the top', where selective and macromolecular mass transport from the pulp forms an enriched froth. But in commercial operations the concentrate (floated) is only a certain amount of froth effectively removed from a cell. Recent attempts at flotation process simulation from kinetic parameters provided by laboratory scale batch cells have shown limited application as, in practice, operators remove only the amount of mass that they judge convenient in any stage of the process.

Submechanisms

Flotation – Capture and mass transfer from slurry phase to froth phase

In this first component, by using hydrophobic differences between particles that can be macroscopically evaluated by the 'flotation kinetic' and also by the 'selectivity', it becomes possible to study and to simulate the natural flotation phenomenon (or only 'froth flotation') and its commercial operations scale-up, referred to as rougher.

Concentration – Macromolecular mass transfer from froth phase to water

Concentration is related to cleaning froth of less hydrophobic particles (gangue and lower-grade intermediate products) and producing a concentrate. In this second component (that has a possibility of being macroscopically evaluated by 'selectivity') one can try to study and to simulate the concentration phenomenon, referred to as 'cleaner'. In composite flotation processes (by steps), the concentration can be studied as a gaseous macromolecular absorption process (froth and water), countercurrent with the desorption initiated in natural flotation (a double mechanism). The chemical engineering operations are evaluated through the 'equilibrium curve' and the 'operation line' (Treybal, 1970).

SLURRY PHASE: NATURAL FLOTATION

Batch operations

Two main mechanisms for a batch system are generally used, as occurs in some laboratory experiences where, generally, the concentrate is removed from the cell at a different moment where the mass is floated (batch). In the case of concentrate removal at the same time as the mass is floated (semi-batch), the concentration is a simple metallurgic consequence of natural flotation phenomenon and, conceptually, the real mass concentration phenomena and natural flotation are the same, therefore they lead to the same macroscopic results.

Mass transfer by froth flotation

An illustration that could help to understand the mass transfer in batch froth flotation in a laboratory cell is shown in Figure 2. The concentration (average grade) of the interest substance falls from a to r , in a total time t . The concentrate is removed by means of froth layers with decreasing grades in function of time, where highly hydrophobic – or richer – particles float at a fast speed, from the richest layer, grade c_1 , in time t_1 , until arriving at the last layer c_n of grade near to a , in time t . The final concentrate possesses an average grade c , between a and c_1 . A basic concept in mass concentration process can be understood as follows: the attainment of aliquot c_n , with grade inferior to a , would lead to deconcentration of the concentrate. The *modelo operacional* establishes that the fraction c_n does not have to be less than a . This is validated for rougher flotation and the equilibrium value for this subprocess will be established taking in account this important definition.

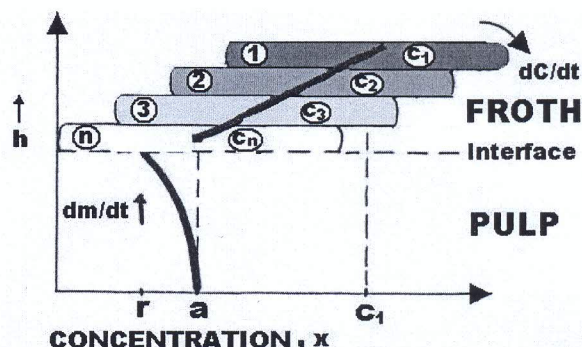


FIG 2 - Mass transfer in batch froth flotation in a laboratory cell. The average grade of the interest substance falls from a to r , in total time t .

Continuous operations

The short circuit factor can be eliminated by using a certain number of cells in a series in a flotation bank, and a residence time not less than the minimum required by the natural phenomenon. It can be expected that the interface substance concentration represents the pulp phase average grade due to agitation.

The equilibrium approach mechanism from pulp to froth phase is illustrated in Figure 3. The steep curve represents the speed of the floated mass: dm/dt in a batch operation, and the curve dC/dt corresponds to the concentrate production speed in a continuous operation. Real flotation follows a different profile of speeds to those observed in the laboratory in a batch test (the times are different). The symbols used are:

- t_{in} = minimum natural flotation time in batch system (laboratory)
- t_f = minimum real flotation time, in continuous operation
- t_r = effective residence time in plant
- R = substance x recovery in concentrate, per cent
- R_{oo} = equilibrium recovery, when the net mass transfer is stopped, per cent

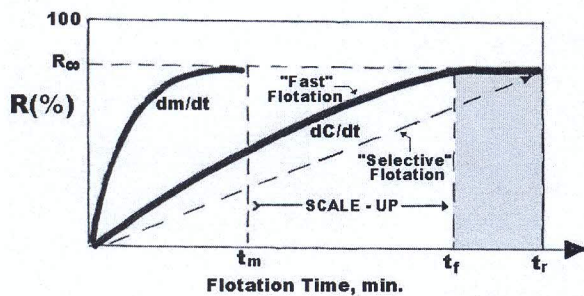


FIG 3 - The mechanism of equilibrium approach from the slurry phase to the froth phase. The steepest curve represents the speed of the floated mass: dm/dt in a batch operation, and the curve dC/dt corresponds to the concentrate production speed in a continuous operation.

In continuous operations, the mass transfer ceases in the time t_r . The shading space (Figure 3), equivalent to $(t_r - t_f)$, corresponds, in practice, to the last cell of the rougher flotation bank, when the circuit has a good operation performance and where the white froth is characteristic of the natural flotation process exhaustion. Scale-up being t_m to t_f is basic in the sizing of flotation circuits. The difference $(t_r - t_f)$ is used normally to favour the selectivity of the froth layer, distributing the concentrated layer throughout all flotation banks with a float mass no more than necessary, as illustrated in the dotted line of Figure 3. Natural flotation is the preference for the pulp phase, where the first hydrophobic particles separation is made in rougher stage. In the cleaner steps, everything is (or already was) froth and the cleaning process of this froth takes control of the process (selectivity in the froth = concentration). Generally, for rougher flotation, the mechanical cells are more adequate, reducing the residence time in the slurry phase and maximising the bubble/particle contacts.

FROTH PHASE – CONCENTRATION

Concentration mass transfer

A macromolecular mass transfer takes place in the froth phase, for both directions in the vertical axle, in accordance with the hydrophobic gradient and, therefore, in accordance with the concentration of the individual grade of particles between the

interface and the froth layer surface. The concentration process, by means of the laundering of froth with water can be effected in continuous form (continuous contact), keeping the phases distant from the equilibrium point. This can be executed, for example, in a deep froth layer with high residence time, removing only the superior layer in continuous flow (column cells). The froth layer is permanently washed with water (in a fine spray form) whose return for the pulp phase drags low hydrophobic particles that had been floated in the rougher stage and that contaminate the concentrate, reducing its global grade. The conventional mechanical cells are accommodated in banks where, in case they require the use of enough flotation time ($t_r > t_f$), they can operate in a fractional way, reaching results next to their equilibrium in the last cell of the bank when they are correctly operated. In industrial practice, the cleaner's flotation banks are accommodated in countercurrent stages, where the laundering water moves in the opposite direction from the main froth flow, draining (absorbing) undesirable particles. This staged disposal speeds up the mass transfer in the froth phase, as demonstrated in chemical operations (molecular). The collection of the superior froth layer in low mass flow will involve high concentrated grade, however recovery will be low.

CONTACT AMONG PHASES

Batch operations

The discontinuous processes executed in a laboratory are generally classified in the rank of batch operations. It's characteristic of not being at a steady state that concentrations in any point of the equipment are a time function. Depending on the way froth is removed, it can be called batch, with discontinuous withdrawal of froth, or semi batch, keeping a continuous flow of froth removal. Normally, if relating to the unsteady state, the most common experience is batch, where the concentrate is removed in time intervals, by small layers, creating a series of aliquot to be chemically analysed. With these results, it is possible to trace: the Kinetic Curve, the Selectivity Curve (Figure 4) and the Concentration Equilibrium Distribution Curve. These are the macroscopic elements used by the 'Modelo Operacional'.

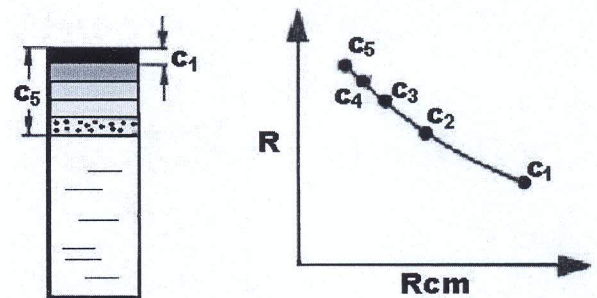


FIG 4 - Selectivity curve. The mass concentration ratio, R_{cm} , is defined. It expresses the mass ratio of feed to concentrate: $R_{cm} = A/C$. By plotting R versus R_{cm} , one obtains the selectivity curve.

Continuous operations

Continuous contact equipment

The hydrophobic equilibrium deviation (the gradient that promotes the transport) is deliberately kept between pulp and froth phases, establishing total continuity of the flows in either steady state, or feeding, concentrated and tail flows. Column cells are typically continuous contact equipment. The residence time depends on the pulp volumetric fed (Q) and the available volume (useful volume) of the flotation cell (V):

$$t_r = V/Q$$

However, the selectivity of one conventional mechanics cell normally will be inferior to one gotten in a discontinuous laboratory cell, due to 'short-circuiting' mass (material that goes directly to the tail, without having a chance to participate in the air bubbles 'capture' mechanism). Therefore, for the same amount of floated material in both systems, for an equivalent time, an inferior metallurgic recovery in the continuous cell and a low grade of concentrate is expected, compared with the laboratory observed values, harming the scale-up. Furthermore, the hydraulic conditions of mass distribution in the product (SPLIT) generally do not allow reaching of the necessary R_{cm} with only one mechanic flotation cell.

The column cells possess more operating resources, because the residence time is sufficiently high (20 to 30 minutes in the industrial practice). The concentrate split allows the reaching of R_{cm} values next to two, or up to 50 per cent of the feed can move to the concentrate flow, depending a little on its H/D (length to diameter) relation.

Fractional contact equipment

To reduce short circuit risks and to reach minimum flotation time t_r , flotation banks with four or more mechanical cells in a series are used. The intermediate concentrates combination is equivalent to the total stage concentrate and the last cell tail corresponds to the joint system tail. A convenient flotation cells number, connected in a series, eliminates the short-circuit load problem, improving the process selectivity. The disadvantage attributed to this equipment is the need for bigger area in commercial operations, despite the fact that it favours the soft and selectivity distribution of the concentrate mass flow. However, bigger models of mechanical cells have attenuated this problem.

EQUILIBRIUM AND DRIVING FORCES

Batch equilibrium

When equilibrium between phases is established, the relative mass transfer ceases. Consider a pulp with average grade ac , as illustrated in Figure 5. As it creates the required physical and chemical conditions for natural flotation (liberation, reagents, etc), when incorporating the air flow, some particles are quickly transferred to the froth, at the speed dm/dt . A small quantity of this mass comes back to the pulp with the drained water. By the hydrophobicity potential that stimulates the transfer, the total froth layer (concentrate) reaches its equilibrium at an average grade c^* . The froth phase, in equilibrium, internally possesses its proper uniformity (or hydrophobic organisation) in the layer's vertical axle. The pulp phase, by agitation, keeps its uniformity based on the total homogeneity of the phase, with average grade r^* . In a steady state, for $t_r > t_f$, the dynamic balance is also achieved. With another pulp of grade ac higher than illustrated in the example, the dynamic equilibrium would be reached with new values of c^* and r^* , also higher than the observed ones in the previous equilibrium condition. In this way, we can plot an equilibrium distribution curve (or equilibrium curve) for the froth/pulp system. For rougher flotation, what is really attained in the laboratory is the equilibrium point, where c^* and r^* grades are defined from other macroscopic experiences (kinetic and selectivity curves) that will determine the rougher equilibrium recovery (R_c), when the last concentrated aliquot c_n possesses the same grade as the feeding a .

Continuous equilibrium

In a continuous process, due to particle transfer between slurry to froth, the concentration in each phase is changed when the pulp flow advances through the equipment (in a batch process the

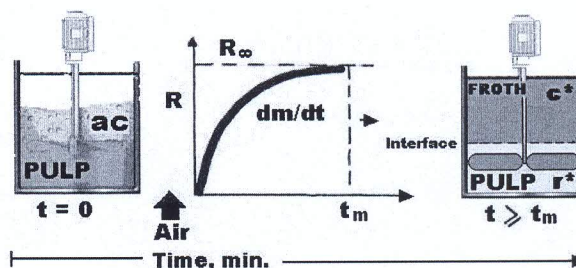


FIG 5 - Dynamic equilibrium between phases in a batch system. The froth phase, besides being in equilibrium, internally possesses its proper uniformity (or hydrophobic organisation) in its vertical axis. The slurry phase, by agitation, keeps its uniformity based on the total homogeneity of the phase, with average grade r^* .

concentration changes throughout the time). The average concentration of evaluated substance x in any point of the froth possesses an average grade of c in the mass flow, that falls for c_i in the interface. In the slurry, the flow concentration that continuously enters the cell falls from r to r_i in the interface, becoming very close to these values in the last cell in flotation bank, when $t_r > t_f$. The values of r and c are not equilibrium values. Concentrations r and c are related in a distinct way between the two phases, by means of the hydrophobic potential between them (delimited for an interface) that is the real driving force of the macromolecular transfer between the phases.

In steady state operations, for the molecular scale, the existence of an equilibrium state in the interface was proven experimentally (Bird *et al*, 1960). But for high speed particle transfer, as could happen in continuous contact columns, some distance of the equilibrium in the interface zone would occur. The *Modelo Operacional*, analogous to what was introduced for the flotation process, relies on the values of r_i and c_i being in equilibrium, given by the equilibrium curve experimentally acquired, when the $t_r > t_f$ condition is fulfilled. Practically, the adjusted residence time of the circuit and the froth layer stability, contribute to the interface to reach and to keep the equilibrium conditions between the phases, allowing that:

$$c_i = c^* \text{ and } r_i = r^*$$

Driving forces and operation line

Consider an operation where it is necessary to place a pulp of r grade in contact with a concentrate of c grade, where:

$$ac = c + r$$

A global expression of the contact would be:

$$c + r + ar \rightarrow c^* + r^*$$

For time $t > t_m$ (batch) or $t_r > t_f$ (continuous) the latter expression could occur. In Figure 6, the shifting of the line of degree of the equilibrium is illustrated, where it finds the process feeding and the operation line. This representation is made by using the equilibrium curve obtained in the laboratory.

Point **P** (r, c) shows the global mass contact between the two phases (in a steady or an unsteady state). Point **E** (r^*, c^*) corresponds to an identical hydrophobic potential between the phases, where the net transfer flow finishes. This effect is easy to imagine in laboratory cells. In continuous processes, in special cases, it is possible to find the physical equilibrium perception in the last cell of a flotation bank, as long as the minimum residence time is guaranteed. For the same contact area, the mass flow from **P** to **E** can be written as:

$$N_x / \text{area} = K_p (r - r^*) = K_E (c^* - c)$$

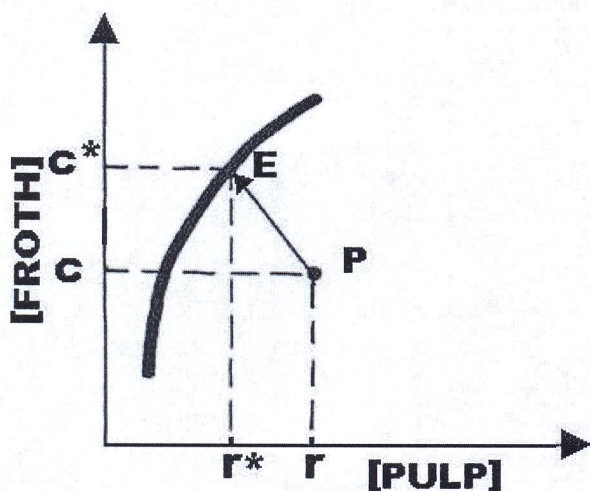


FIG 6 - Driving forces and operation line. This representation is made using the equilibrium curve obtained in laboratory.

where the direction of the flow is:

$$(c - c^*) / (r - r^*) = - (K_p / K_E)$$

where:

K_p, K_E = transfer coefficients in the slurry and froth phase, respectively

$-(K_p / K_E)$ is equivalent to the slope of line **PE**, the operation line of this process

If we knew the mass transfer coefficients in each phase, it would be possible to determine the conditions of the interface and the mass flow N_x in one determined instant, either graphically, representing the operation line **PE**, or analytically, with a mathematical expression for the equilibrium curve (found experimentally) of the type: $c^* = f(r^*)$.

For a continuous process, the concentration inside each phase moves from where the pulp flow advances to the flotation bank. The maximum value, c_n , from the global mass flow of c is normally found in the first froth layer removed from the first cell of the flotation bank. The pulp concentration reaches its minimum value in r^* , together with the mass transfer ending in the final tails flow in the last cell of the bank. Similarly, in the batch system, as in the laboratory, the concentration of each phase moves as a function of the flotation time, until reaching the equilibrium in the time t_m .

MASS TRANSFER FLUX

In accordance with operational model first law (Yovanovic *et al*, 1988, 1991, 1992, 2004) it is possible to define the product mass flows from a project to a commercial plant (by means of the **Rcm**). In industrial practice the flows can be duly sampled and analysed, and the optimised circuits would always operate in conditions next to the equilibrium curve. The equation that defines the mass transfer flow of x is:

$$N_x = K_p \times A \times (r - r^*) = K_E \times A \times (c^* - c)$$

The macroscopic factors that affect the N_x flow are as follows.

Transfer coefficients (K_p, K_E)

Agitation, in slurry phase, speeds up mass transfer. Slurry density and viscosity cannot be very high. For flotation columns, the great froth height creates an accented pressure difference

between the point where the air bubble is fed and the froth layer surface. This increases the physical and hydrophobic gradient of the mineralised air bubble, which is the particles way of transport, increasing its ascension speed. This condition is not necessarily beneficial since the process as a whole has two ways, considering the macromolecular absorption (draining), and this last one, which is harmed by linear high transfer speed, having demanded a larger amount of laundering water to keep the positive bias binding in the column.

Area

The contact area for the desorption process (slurry to froth) is distributed in thousands of bubble/slurry contacts. However, for froth draining (absorption), the contact area possesses enormous importance. As has been indicated, the continuous contact column process has smaller draining areas than conventional cells, delaying the absorption process. This is compensated for by a longer residence time (column height) in the froth layer and greater laundering water flow. The concentrated mass SPLIT is wrong for low relations of column H/D. The conventional cell, for example, needs a minimum number of cells in a series to reach the quantity of concentrated mass that the process requires. Industrial column cells present difficulties in the last cleaning steps, where they may require high mass SPLIT.

Gradients ($r - r^*$) and ($c^* - c$)

We have expressed the gradients as a function of substance concentrations and not in terms of its hydrophobic differences. Moreover, we must remember that the particle is not put into motion by itself, but is transported via the air bubble. The maximisation of the enrichment quotient (c/r) is the main process objective, and kinetic and selectivity studies mention this gradient maximisation. Adjusted reagents, slurry pH, liberation degree, particle size and others, are mainly natural flotation relative aspects (in the slurry phase). In the froth phase, the circuit is organised with countercurrent steps and, in some cases, with a new liberation point (regrind) that favours the concentration gradient.

MACROSCOPIC BALANCE AND STEPS NUMBER

Rougher flotation

In Figure 7, a rougher flotation operation is represented in opened circuit (without a circulating load) in accordance with the proposal analogy. The represented rougher flotation circuit corresponds to a continuous steady state, of one-step, in parallel flux in the same one felt. The material feeding grade, represented by **a**, corresponds in practice to the first pulp submitted to the flotation process. Liberation characteristics, reagents dosage and other natural flotation conditions will define the rougher equilibrium point, when $R_e = R_{00}$, R_e being the final accumulated recovery at the moment when the last concentrate aliquot removed from the cell is identical to the feeding grade, taking into account that $t_r > t_f$.

The values r^* and cr^* are measured experimentally. From Figure 7, we can make the following analysis:



where:

A, R and CR correspond to mass ore flows (mass/time) in feed, tail and rougher concentrate, respectively

The interest substance distribution for a 100 base [100] in the feeding is:



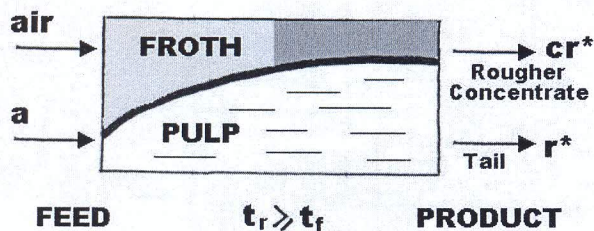


FIG 7 - Analogical representation of rougher flotation. The represented rougher flotation circuit corresponds to a continuous steady state of one-step.

where:

[A] = A × a

[R] = R × r*

[CR] = CR × cr*

Substituting the previous values, it results that:

$$(R + CR) \times a = r^* \times R + cr^* \times CR$$

$$R \times (a - r^*) = CR \times (cr^* - a)$$

This expression corresponds to a straight line, the rougher flotation operation line, whose slope is $-(R/CR)$, when adjusting the driving forces on the flow direction. The operation line PE, that passes through the feed point P (a, a) and the exit point E (cr*, r*) is not of much utility for the rougher operation, since the point E can be defined in the laboratory, by means of kinetic and selectivity mechanisms. The rougher must be operated in the equilibrium point.

Concentration in cleaning steps

The concentration equilibrium curve can be traced with experimental data. When the equilibrium point is obtained by rougher flotation it is possible to follow the procedure indicated in Figure 8 to trace the equilibrium curve. After joining froth (rougher concentrate) in equilibrium conditions, it must be reconditioned with clean water and floated again until the exhaustion and extraction of aliquot and measurement of c₁* and r₁*. Then it must be re-floated twice following the same procedure as illustrated in Figure 8. The concentration circuit, or cleaner, is fed with product cr* (equilibrium rougher concentrate) and the metallurgic objective of the process is to obtain a final concentrate, c, at minimum cost with maximum possible recovery. A cleaner operation is represented in Figure 9, in countercurrent, in accordance with the macromolecular transfer model. The flow cr* represents the equilibrium rougher concentrate, and the flow rc is the cleaner circuit tail.

- Mass balance: CR + water = RC + C
- Substance distribution: (RC + C) × cr* = rc × RC + c × C
- Also, RC × (cr* - rc) = C × (c - cr*)
- Operation line: (c - cr*) / (cr* - rc) = RC / C
- Recovery: Rc = [(cr* - rc) (c)] / [(c - rc) (cr*)]
- Total plant recovery: RT = Re × Rc

where the factor (RC/C) is the operation line slope and flows in a positive direction and passes through the points A (cr*, rc) and P (c, c), as illustrated in Figure 10. For a cleaner stage in equilibrium, the point A can be graphically calculated following straight lines PE and EA. For a high grade of c, in the final concentrate, there will be a corresponding high grade of rc, and a smaller recovery in the concentration circuit.

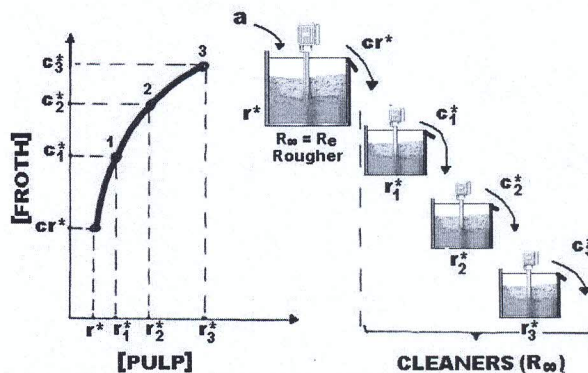


FIG 8 - Concentration equilibrium curve. When rougher flotation obtains its equilibrium point, it is possible to follow the procedure indicated in Figure 8 to obtain the concentration equilibrium curve.

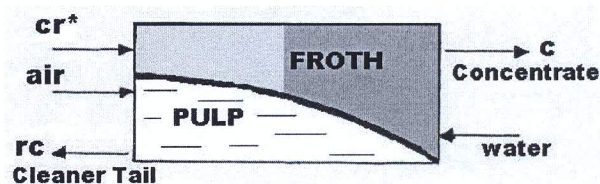


FIG 9 - Analogical representation of cleaner flotation.

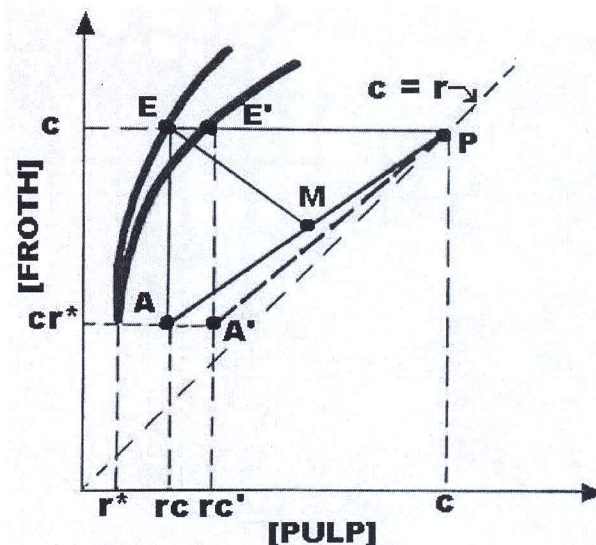


FIG 10 - Operation line in one step concentration process. For a cleaner step, in equilibrium, the point A can be graphically calculated following straight lines PE and EA. For a high grade of c in the final concentrate, there will be a corresponding high grade of rc, and a smaller recovery in the concentration circuit.

The final value of c does not need to be the equilibrium value, but this can be achieved in the following conditions using a convenient number of steps. On the other hand, few selective processes (liberation problems, low hydrophobicity, etc) possess different equilibrium curves, as illustrated by the less sloped curve in Figure 10, producing a bigger grade rc' (in the new equilibrium point E') and a lesser Rc' recovery. The operation

line **AP** is a graphical representation of the process mass balance, which goes from one edge of the equipment to the other. Point **M** represents the global concentrations of the two flows that cross in the step and possess a driving force that takes this stage to the equilibrium. In less selective processes, or when it is necessary to get a high grade concentrate, it is convenient to divide the concentration process in to countercurrent steps, as illustrated in Figure 11.

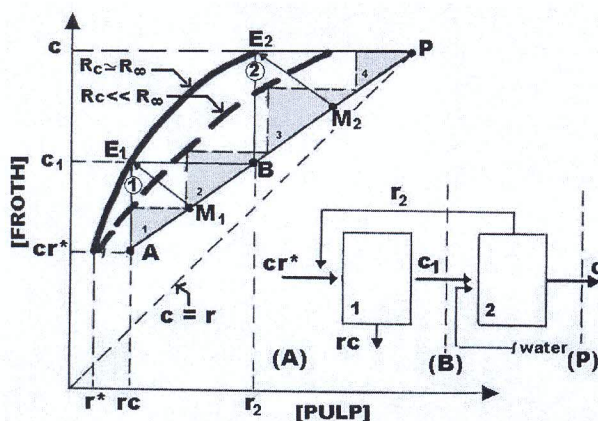


FIG 11 - Ideal concentration steps. Knowing the value of cr^* and Re (equilibrium rougher concentrate) the project designer can define the values of c and R_T that would justify the process. The value of rc can be calculated. The operation line is obtained between $A(cr^*, rc)$ and $P(c, c)$. The graphical movement advances from AE_1 , E_1B , BE_2 and E_2P until reaching the value of intended c .

APPLICATIONS

- Knowing the value of cr^* and Re (equilibrium Rougher concentrate) the project designer can define the values of c and R_T that would justify the process. The value of rc can be calculated by the last two equations. The Operation Line is traced between $A(cr^*, rc)$ and $P(c, c)$. The graphical movement advances from AE_1 , E_1B , BE_2 and E_2P until reaching the value of intended c .
- Circuits badly operated, with a low equilibrium approach, as illustrated in the dotted lines from Figure 11 (for $Rc \ll R_{\infty}$), require more steps to reach the same objective (four in this case). The dotted curve corresponds to the real contact curve between phases (real plant operation), with results in the separation of the equilibrium curve ($Rc = R_{\infty}$) in accordance with the plant operation quality or by an inadequate layout, for example, a lack of cells in the bank.

- Consider a plant operating with low efficiency, as illustrated in the dotted curve of Figure 11 (four stages). If we send part of the pulp flow to some point next to **B** – for example, the second cleaner concentrate – then installed new equipment in parallel to the circuit, and operated it correctly, using to advantage all the driving force (passage **B-E₂-P**), it would give a false impression that the tested equipment is more selective than the equipment of the current industrial circuit. One could wrongly conclude that this equipment could substitute two current concentration steps. What is really the case is that the plant is poorly operated, far from the equilibrium curve, and needs to be optimised before any evaluation of this type.

CONCLUDING REMARKS

A research group was recently created in Brazil, supervised by the UFMG (Universidade Federal de Minas Gerais), where the authors are involved. The aim of this group is to demonstrate and develop this new approach for several units operations (crushing, grinding, flotation, etc). During 2005, as the work progresses, the group hopes to reach important findings in order to challenge some of several minerals processing paradigms.

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